**Homework #8: From Language to Logic**

# Problem 4

## Problem 4a

The knowledge base is

First, we apply the property that , which allows us to write the knowledge base as

Distributing the negation connectives on A and B we have

We can remove the conjunction operator to yield the equivalent group of atomic expressions

Finally, we apply the same identity as in the first step to convert back into implication connectives, which then allows us to express the knowledge base into Conjunctive Normal Form:

To apply modus ponens to the above expression of the knowledge base, we accept the conditional statement , and that the antecedent holds, such that we can infer .

## Problem 4b

The knowledge base is

Once again applying the identity , we have

Distribution of the third rule’s disjunction operation on A and C and applying the identity once again gives us the knowledge base expressed in CNF:

From the first two rules of the knowledge base, we obtain

We can combine this result with the fourth rule so that we have

Finally, applying that result to the third rule allows us to derive D directly:

# Problem 5

## Problem 5b

In order to prove that the resulting set of 7 constraints is not consistent for any finite, non-empty model, we can consider a sequence of elements in a domain of size . For every in the sequence of elements, we say that is a successor of . Then, the fifth constraint dictates that must also be larger than , and the sixth constraint says that any is greater than .

However, since there are elements in the sequence and only values specified in the domain, this means that there must be at least one pair of numbers that are equivalent, i.e. . However, the transitive sixth constraint says that , while the new seventh constraint says that a number cannot be larger than itself. **Therefore, these two constraints contradict one another in this case, which means the set of constraints is inconsistent.**